

DYNAMICS AND OPTIMAL HARVESTING OF PREY-PREDATOR FISHERY
MODELS IN THE PRESENCE OF A TOXICANT USING PONTRYAGIN
MAXIMUM PRINCIPLE

ANG TAU KEONG

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This work is humbly dedicated to all of my valuable treasures in life:

My beloved parents and family.

Thanks for your endless support!



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ABSTRACT

In this thesis, several harvested fishery models using various types of harvesting strategies including the common harvesting, independent harvesting and nonlinear Michaelis-Menten harvesting functions are presented and analyzed. Besides that, most of the models are taking the existence of a toxicant into consideration, either in an anthropogenic or a self-produced form. In order to account for the intraguild predation interaction that occurs in marine ecosystems, three species fishery models are also studied by modeling the carrying capacities of both intraguild prey and predator fish as time-dependent variables. The idea of variable carrying capacity tends to describe the fish population dynamics in a varying environment by assuming their growth to be dependent on the environmental resource availability. In the context of resource management, an optimal harvesting policy is derived for the proposed models that aims to attain an optimal and sustainable yield of harvesting fisheries. Recent studies reveal that the study of harvesting alone is not sufficient to provide qualitative insights into the intrinsic behaviors of marine fish populations subject to harvesting. In fact, some of the important elements such as the presence of toxicants and intraguild prey-predation, as well as the interplay of the population with surrounding environment, are often neglected in most of the current studies. Therefore, the main contribution of this thesis is to generate insights into the dynamical behaviors of fishery models that account for the combination of the existence of harvesting, toxicant, as well as intraguild predation interaction. The dynamical properties discussed in this thesis give better understanding of the maximum thresholds of harvesting before the fish populations are driven to extinction. This thesis serves as a platform to investigate the optimal tradeoff of harvesting fisheries.

ABSTRAK

Dalam tesis ini, beberapa model perikanan yang menggunakan pelbagai jenis strategi penuaian termasuk penuaian bersifat umum, bebas dan "Michaelis-Menten" telah dibentang dan dianalisis. Selain itu, kebanyakan model mengambil kira kewujudan bahan toksik, sama ada dalam bentuk antropogenik atau yang dihasilkan sendiri. Dalam usaha untuk mengambil kira interaksi rantaian makanan pemangsa yang berlaku dalam ekosistem marin, model melibatkan tiga spesies ikan telah dikaji dengan memodelkan kapasiti tampungan mangsa dan pemangsa sebagai pembolehubah bergantung pada masa. Idea pembolehubah kapasiti tampungan cenderung untuk menggambarkan dinamik populasi ikan dalam persekitaran yang berbeza-beza dengan menganggap pertumbuhan mereka adalah bergantung pada ketersediaan sumber di sekelilingnya. Dalam konteks pengurusan sumber, dasar penuaian optimum diperolehi daripada model yang dicadangkan bertujuan untuk mendapatkan hasil penuaian perikanan yang optimum dan mapan. Kajian baru-baru ini mendedahkan bahawa kajian yang melibatkan penuaian sahaja tidak mencukupi untuk memberi pandangan kualitatif ke atas tingkah laku intrinsik populasi ikan marin yang tertakluk kepada proses penuaian. Malah, beberapa elemen yang penting seperti kewujudan bahan toksik dan rantaian makanan pemangsa, serta interaksi antara populasi dengan persekitarannya, sering diabaikan dalam kebanyakan kajian semasa. Oleh itu, sumbangan utama tesis ini adalah untuk menjana pandangan ke atas tingkah laku dinamik model perikanan yang menyumbang kepada gabungan kewujudan penuaian, toksik dan juga interaksi rantaian makanan pemangsa. Ciri-ciri dinamik yang dibincangkan dalam tesis ini memberi pemahaman tentang ambang penuaian maksimum agar populasi ikan yang dituai tersebut tidak ke arah kepupusan. Tesis ini berfungsi sebagai platform untuk menyiasat optimum "tradeoff" untuk menuai sumber perikanan.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	- Rate of change of prey fish due to prey-predation
a_0	- Coefficient of increase of prey in the absence of other species
b	- Coefficient of toxicity on prey fish
b_0	- Coefficient of decrease of prey due to interaction
c	- Rate of change of predator fish due to prey-predation
c_0	- Coefficient of increase of predator due to interaction
c_1	- Constant harvesting cost per unit effort of prey fish
c_2	- Constant harvesting cost per unit effort of predator fish
d	- Coefficient of toxicity on predator fish
d_0	- Coefficient of decrease of predator in the absence of other species
d_1, d_2, d_3, d_4	- Positive constants
E	- Combined harvesting effort
E_1	- Distinct harvesting effort applied on prey fish
E_2	- Distinct harvesting effort applied on predator fish
e_0	- Density of prey required to maintain a single predator
f_1, f_2, f_3, f_4	- Variable functions
g	- Growth rate of biotic resource
H	- Hamiltonian function
H_1	- Harvesting regime of prey fish

H_2	- Harvesting regime of predator fish
h_1	- Constant harvesting rate of prey fish
h_2	- Constant harvesting rate of predator fish
J	- Jacobian matrix
K_1	- Environmental carrying capacity of prey fish
K_2	- Environmental carrying capacity of predator fish
m	- Accessibility of prey fish towards biotic resource
n	- Accessibility of predator fish towards biotic resource
P_1, P_2, P_3, P_4	- Equilibrium points
q_1	- Catchability coefficient of prey fish
q_2	- Catchability coefficient of predator fish
r_1	- Growth rate of prey fish
r_2	- Growth rate of predator fish
s_1	- Constant selling price of prey fish
s_2	- Constant selling price of predator fish
t	- Time
u	- Consumption rate of resource by prey fish
V	- Lyapunov function
v	- Consumption rate of resource by predator fish
X, x	- Prey biomass densities
Y, y	- Predator biomass densities
Z, z	- Resource biomass densities

$\alpha, \hat{\alpha}, \gamma, \eta$	
$\hat{\eta}, \theta, \bar{\theta}, \mu$	- Dimensionless parameters
$\nu, \xi, \sigma, \tau, \chi$	
ω	
$\beta, \tilde{\beta}$	- Dimensionless harvesting parameters of prey fish
$\delta, \tilde{\delta}, \bar{\delta}, \bar{\delta}$	- Dimensionless toxicant parameters of prey fish
$\hat{\delta}$	- Annual fishery discounting factor
$\varepsilon, \tilde{\varepsilon}, \bar{\varepsilon}$	- Dimensionless toxicant parameters of predator fish
λ	- Eigenvalue
$\lambda_1, \lambda_2, \lambda_3$	- Adjoint variables
π	- Economic rent
$\rho, \tilde{\rho}, \dot{\rho}$	- Dimensionless harvesting parameters of predator fish
ψ, ψ_1, ψ_2	- Switching functions
ϕ	- Dulac function
BTP	- Biotechnical productivity
HB	- Hopf bifurcation
TB	- Transcritical bifurcation

LIST OF PUBLICATIONS

Journal:

- i. Ang, T. K., Safuan, H. M. and Kavikumar, J. (2018). Dynamical behaviours of prey-predator fishery model with harvesting affected by toxic substances. *Malaysian Journal of Industrial and Applied Mathematics*. 34(1), 143-151.
- ii. Ang, T. K., Safuan, H. M. and Kavikumar, J. (2018). The impacts of harvesting activities on prey-predator fishery model in the presence of toxin. *Journal of Science and Technology*. 10(2), 128-135.
- iii. Ang, T. K., Safuan, H. M., Sidhu, H. S., Jovanoski, Z. and Towers, I. N. (2019). Impact of harvesting on a bioeconomic predator-prey fishery model subject to environmental toxicant. *Bulletin of Mathematical Biology*. 81(7), 2748-2767.
- iv. Ang, T. K. and Safuan, H. M. (2019). Harvesting in a toxicated intraguild predator-prey fishery model with variable carrying capacity. *Chaos, Solitons and Fractals*. 126, 158-168.
- v. Ang, T. K. and Safuan, H. M. (2020). Dynamical behaviors and optimal harvesting of an intraguild prey-predator fishery model with Michaelis-Menten type predator harvesting. *Manuscript submitted for publication (Under Review)*.

Book Chapter:

- i. Ang, T. K., Safuan, H. M., Roslan, U. A. M. and Mohd, M. H. (2019). Optimal harvesting regions of a polluted predator-prey fishery system. In: *Dynamical Systems, Bifurcation Analysis and Applications (DySBA 2018)*. Springer Proceedings in Mathematics and Statistics, vol 295. Singapore: Springer. 15-29.

CHAPTER 1

INTRODUCTION

In this chapter, there are six sections explaining the rationales of this research. Section 1.1 provides the background of the research while Section 1.2 discusses the statement of the problem. The research objectives are presented in Section 1.3. Section 1.4 explains the scope of the study and Section 1.5 highlights the significance of the study. Finally, Section 1.6 presents the overview of this thesis.

1.1 Research background

Fishery resources from the seas and oceans are undoubtedly a vital source of animal protein to human being. In 2016, approximately 87% of the total fish caught are used for the purpose of human consumption while the other 13% are used for non-consumption purposes (FAO, 2016). The rising demand for seafood provokes the process of exploitation of fisheries, causing the severe depletion or eventual extinction of some marine species such as Pacific Tuna (Burgess *et al.*, 2017) and Snapper (Guardia *et al.*, 2018). Generally, there are two types of over-fishing in fishery science: recruitment over-fishing and growth over-fishing (Hernandez *et al.*, 2016). Recruitment over-fishing refers to the situation when an adult fish population is harvested beyond the level that it can be replenished again while growth over-fishing is the premature harvesting on the juvenile fish population. In fact, neither over-fishing nor under-fishing is rational as both of them bring about some adverse impacts to fishery conservation (Zhou *et al.*, 2015). This is because over-fishing can cause a severe decline in fish population density but under-fishing can pose a loss to human

society. Thus, implementation of a balanced harvesting strategy is essential to prevent over-exploitation of fishery resources and simultaneously ensure the continuous flow of benefits to human society.

Besides the issue of over-exploited fisheries, concerns are mounting about the presence of toxicants in aquatic environment. A handful of fishery studies, with the supports of laboratory data has been done to study the human exposure to aquatic toxicant - methylmercury, through the consumption of seafood (Li *et al.*, 2016b; Schartup *et al.*, 2018; Sunderland *et al.*, 2018). From their research, methylmercury is an organic form of mercury released into aquatic environment through human activities. This anthropogenic toxicant is the only form of mercury that can bioaccumulate and biomagnify in marine food webs especially in the tissues of predatory fish species such as shark, cod, tuna, salmon and swordfish. Continuous human exposure to this environmental toxicant through ingestion is known to have some detrimental effects on neurodevelopment and cardiovascular health. Furthermore, it is reported in the research of Schartup *et al.* (2019) that overfishing has a close relationship with the toxicant concentration in marine fish species. This is because the over-exploitation of fisheries can alter the structure of marine food webs, causing the dietary shifts in predatory fish. For instance, the methylmercury concentration in Atlantic cod was lower in 1970s as a consequence of the over-harvesting of herring fish that led to the dietary shifts of Atlantic cod to feed on Clupeidae fish.

Fluctuations of the number of marine fish species due to the presence of harvesting activities and toxicants can definitely pose some undesirable effects on the entire aquatic ecosystem. Therefore, there is a substantial growth of global concerns on the analysis of fish population models in order to examine the dynamical behaviors among fish with the goal to return the number of fish to its satisfactory extent. In order to delve into the fish population ecology, the interaction between the fish species with each other as well as their interaction with surrounding environment are some of the underlying concepts. Prey-predation is a predominant interaction that affects fish population dynamics (Whipple *et al.*, 2000). Therefore, prey-predator fishery models

are greatly studied by applied mathematicians and ecologists. The studies on prey-predator fishery models are still an on-going process as they exhibit a wide range of interesting dynamics such as steady-states, periodic orbits, bistability and chaos.

Intraguild predation is an interaction when both competition and predation occur simultaneously or when a predator competes with its prey to share the same resource (Holt and Polis, 1997). Intraguild predation can be found in marine ecosystems such as Atlantic capelin (Yurkowski *et al.*, 2016), anchovy (Bachiller *et al.*, 2015) and jellyfish (Meyer *et al.*, 2016). It plays an important role in sustaining a balanced marine ecosystem and it might cause a failure in understanding the interplay between fish population dynamics and harvesting if intraguild predation is not taken into account (Irigoien and Roos, 2011). On the other hand, in modeling an intraguild fishery model, treating the environmental carrying capacity as a constant is not often realistic as the population growth and decay of fisheries are greatly altered by the changes in the surrounding environment including the resource availability (Safuan *et al.*, 2013). Hence, modeling an intraguild fishery model incorporating a variable carrying capacity provides a thorough understanding on the links between fish population dynamics and a changing environment.

Although developing more realistic fishery models remains the principal aim among applied mathematicians, studying the proposed models from the bioeconomic perspectives can supplement the studies on optimal management of fishery resources. The derivation of optimal harvesting policy in a commercial fishery helps to determine or decide the optimal tradeoff between economic output, environmental protection and resource sustainability (Belkhodja *et al.*, 2018). In other words, by optimal harvesting policy, the optimal harvesting rate that provides maximum economic profit while not triggering the extinction of any fish species can be determined. A common practice in studying the optimal harvesting policy of commercial fisheries is by applying the Pontryagin Maximum Principle (Jana *et al.*, 2016; Zhao *et al.*, 2017; Bayon *et al.*, 2019).

1.2 Problem statement

In recent years, the excessive and unsustainable exploitation of fishery resources has led to a global issue that the number of fish declines severely. Furthermore, the presence of aquatic toxicants is deteriorating the aquatic environment. Without a rational approach to cope with the issues of over-exploitation and pollution, many drastic problems might appear that can probably affect the daily life of human being. The problems include the global price rise of fish market, extinction of some rare fish species and even the explosion in the entire marine ecosystems.

Most of the fishery models do not account for the combination of the existence of harvesting, toxicant and intraguild predation. Therefore, the motivation of this research stems from the problem of modeling a more realistic harvested fishery system by taking into account the presence of toxicants as well as intraguild predation interaction. Besides that, this research aims to investigate the dynamics of both two species and three species fishery systems when they are subjected to linear and nonlinear harvesting strategies. The reason is that although harvesting on population models are studied extensively, there is limited literature focusing on the independent and nonlinear harvesting strategies on three species fishery models. Finally, from the economic perspective, it is found that there is limited literature discussing the optimal harvesting policy for intraguild models using Pontryagin Maximum Principle. Thus, in this thesis, the optimal harvesting policies of the proposed models are considered mathematically to study the optimal harvest rate of fisheries with a bid to alleviate the problem of over-harvesting.

1.3 Research objectives

The main objectives of this research are to:

1. investigate the dynamics of harvested prey-predator fishery models in the presence of a toxicant with common and independent harvesting strategies;

2. develop intraguild prey-predator fishery models with linear and nonlinear harvesting through variable carrying capacity;
3. analyze the impacts of harvesting on prey-predator fishery models using bifurcation analysis;
4. derive the optimal harvesting policies for harvested and toxicated prey-predator fishery models with linear and nonlinear harvesting.

1.4 Scope of study

For this research, the scope is limited to two dimensional prey-predator and three dimensional intraguild prey-predator fishery models, modeling through ordinary differential equations. The models are formulated in such a way that the functional response of prey-predation obeys a linear function. Moreover, the presence of a toxicant in the fishery models is modeled through an implicit approach. The proposed models are discussed and studied in terms of stability, bifurcation and bioeconomic analysis. In most of the models studied in this research, harvesting parameter is treated as the primary bifurcation parameter to examine the long-time behaviors. The optimal harvesting policies with independent harvesting and nonlinear harvesting strategies are studied based on Pontryagin Maximum Principle.

1.5 Significance of study

This research presents six mathematical models to study the fish population dynamics affected by harvesting activities in the presence of a toxicant. Several harvesting strategies such as common, independent and nonlinear harvesting strategies are taken into consideration. The mathematical and bifurcation results are ecologically meaningful to generate insights into the persistence and extinction behaviors of fish population in real life. The existence of some interesting bifurcations such as transcritical, Hopf and bistability helps to describe the destabilization scenarios in marine ecosystems due to over-harvesting. Most of the intraguild prey-predator models

developed by applied mathematicians aim to study the impacts of resource enrichment instead of harvesting on the population dynamics. Thus, the intraguild fishery models studied in this research would provide initial guidelines to understand how the prey fish, predator fish and common resource populations vary when there exist some harvesting activities. The optimal harvesting policies derived for the three species fishery models could garner an interest in determining the ideal harvesting rate from both mathematical, biological and economic viewpoints to overcome the problem of over-fishing.

1.6 Overview of thesis

The main aim of this thesis is to develop and study fish population models subject to different types of harvesting strategies. The presence of some anthropogenic and self-producing toxicants, as well as the intraguild predation interaction is taken into consideration. This thesis consists of six chapters, including this introductory chapter. The next chapter, Chapter 2 introduces the discovery and evolutionary studies of prey-predator models. Some extensions on prey-predator models such as harvesting, toxicants and intraguild prey-predation are also discussed. Chapter 3 presents the mathematical methods that are utilized to analyze the proposed models in this thesis. Chapter 4 introduces and analyzes four toxicated fishery models where two of them are with common harvesting strategy while the other two are with independent harvesting strategy. Chapter 5 presents three species intraguild fishery models with linear harvesting and nonlinear harvesting, respectively. Lastly, Chapter 6 is about the concluding remarks by summarizing all the main results and recommendations for future work.

CHAPTER 2

LITERATURE REVIEW

This chapter highlights some of the existing literature as an overview to the studies of fishery models. The chapter consists of four sections, starting with a brief introduction of prey-predator models in Section 2.1. Section 2.2 emphasizes the commercial harvesting activities on fishery resource. Section 2.3 explains the influences of toxicants on fish population dynamics while Section 2.4 focuses on the intraguild predation in fishery models. Lastly, a conclusion summarizing this chapter is presented in Section 2.5.

2.1 Prey-predator models

Prey-predator models can be considered as the building blocks of ecosystems to study the interaction between two or more species in nature. The study of prey-predator models is a research area of particular historical and contemporary interest to both ecological theorists and experimentalists. The pioneering work on prey-predator theory was done by Lotka (1925) where he proposed the first model of prey-predator interaction by incorporating the concept of mass action. The proposed model is given as

$$\begin{aligned}\frac{dX}{dt} &= a_0X - b_0XY, \\ \frac{dY}{dt} &= c_0XY - d_0Y,\end{aligned}\tag{2.1}$$

where X and Y are prey and predator biomass densities respectively. Parameters a_0 and d_0 denote the rates of change of prey and predator respectively, in the absence of the other species while b_0 and c_0 indicate the rates of change of prey and predator

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